Interference and Diffraction – Extra Review Packet

I designed this to help those of you who may need a little extra boost in your understanding of how to tackle interference and diffraction problems, in addition to what we've done in recitation. These problems are NOT intended to be practice problems for the exam. If you're just looking to practice or get an idea of what exam problems will be like, then stick with the practice exam, quizzes, homework, and in-lecture examples from Professor Smith.

Some of you have been asking for a way to visualize what's happening. Here are 2 basic but useful YouTube videos. If you want more, I'm confident in your ability to find related information.

http://youtu.be/BH0NfVUTWG4

http://youtu.be/9D8cPrEAGyc

One way to think about interference is to use the chart I gave you in recitation as an overview. Each type of problem is divided into two parts to learn: first learn to use the information in the chart, and then learn the (often geometrical) details of the problem. Also make sure you know the even more fundamental problems about how to make in-phase and out-of-phase waves in general interfere both constructively and destructively (and somewhere in-between). The first half of this packet is organized accordingly.

For diffraction, [watch the videos and come back tomorrow or Wednesday!]

Don't forget you also need to know reflecting and refracting surfaces, mirrors and lenses! Those topics are not included here, but you must study them nonetheless.

Fundamentals of Wave Interference

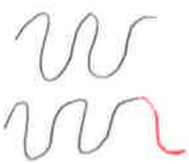
Before we get to the chart, let's make sure we understand the ideas of interference.

If we have 2 waves that begin in phase (rough sketches)

then as long as they both travel the same distance, they will interfere constructively.

maxima meet

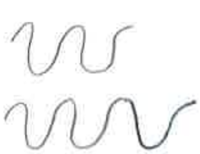
But what if they don't travel the same distance? What if one travels half a wavelength more than the other?



then they will interfere destructively.

maximum + minimum

Now, what if one travels a whole wavelength more than the other?



they will once again interfere constructively

maximum + maximum

This is why we say that <u>if waves begin in phase</u>, then

path length difference $= m\lambda$ constructive

path length difference $= (m+\frac{1}{2})\lambda$ destructive

If you perform a similar analysis for <u>waves that begin completely (π) out of phase</u>, you will see

path length difference $= m\lambda$ destructive

path length difference $= (m+\frac{1}{2})\lambda$ constructive

This is the basis for the chart we made in recitation, and it is also a simplified introduction to the equation

$$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$$

Let's use this equation in a couple problems, but first, let's decompose it briefly.

$$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$$

$$N_2 - N_1 = L\left(\frac{n_2}{\lambda} - \frac{n_1}{\lambda}\right)$$

$$N_2 - N_1 = L\left(\frac{1}{\lambda_{n_2}} - \frac{1}{\lambda_{n_1}}\right)$$

So the number of wavelengths traveled is simply the length divided by the wavelength in the material.

$$N_2 = \frac{L}{\lambda_{n_2}} \qquad \# \text{ wavelengths} = \left(\frac{\text{distance traveled total}}{\text{distance traveled by 1 wavelength}}\right)$$

Then the difference in the number of wavelengths between 2 waves should be a whole number if they begin in phase and end in phase OR if they begin π out of phase and end π out of phase.

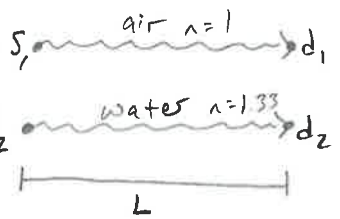
Or it should be a "half number" of wavelengths if the waves begin in phase and end π out of phase OR if they begin π out of phase and end in phase.

But look - this equation does more than any of the equations in our chart. It allows for any phase difference - not just zero or π. The waves could end up $\frac{3}{4}$ of a wavelength ($\frac{3\pi}{2}$ rad) out of phase or 0.277 wavelengths (1.74 rad) out of phase.

Now let's see this equation in action.

Question: Two waves are emitted in phase, but one travels through water and the other travels through air. If detectors for each wave are placed equal distances from the sources, what must that distance be if the wavelength emitted is 390nm (in air) and the waves arrive in phase?

Solution:



$\lambda = 390\,nm$

We can easily set up the right hand side of the equation with this information. As in (dare I say?) most cases, it doesn't matter here which is n_1 and n_2. To make things easier, we'll make the difference positive.

$$N_2 - N_1 = \frac{L}{(390\times10^{-9}\,m)}(1.33-1)$$

Now, what do we do with the left hand side? Well, we want the waves to end up in phase, and we know they began in phase. So how many wavelengths different should their paths be? 0, 1, 2, 3, basically a whole number.

$$k = \frac{L}{(390\times10^{-9}\,m)}(1.33-1)$$

where k is a whole number (use whatever variable you want - it doesn't have to be k)

$$L = \frac{k(390\times10^{-9}\,m)}{1.33-1}$$

This is as much as we can answer, given the problem. Except we can probably say $k \neq 0$ unless we want $L=0$. If the problem asked for the smallest length possible, then we could say $k=1$, so

$$L = \frac{(1)(390\times10^{-9}\,m)}{1.33-1}$$

$$\boxed{L = 1.18\,\mu m}$$

Question: Two light waves of wavelength 430nm in air are emitted in phase, but in different materials. One travels through a material whose index of refraction is $n = 1.37$, and the other travels through a material whose index of refraction is $n = 1.55$. What is the smallest length the waves must travel to be exactly 1 rad out of phase?

Solution: $1\,\cancel{\text{rad}}\left(\frac{1\text{ wavelength}}{2\pi\,\cancel{\text{rad}}}\right) = \frac{1}{2\pi}$ wavelengths is what goes on the left-hand side of our equation

$\lambda = 430\times10^{-9}\,m$

Again, it doesn't matter which is n_2 and n_1.

$$\frac{1}{2\pi} = \frac{L}{430\times10^{-9}\,m}(1.55-1.37)$$

$$L = 1.5\times10^{-5}\,m$$

$$\boxed{L = 15\,\mu m}$$

Notice that we were given the difference in number of wavelengths, but we were given it in radians, so we had to convert. We converted phase difference to wavelength difference.

If we were asked to find the second smallest length, we would remember that they will have the same phase difference if they are out of phase by 1 rad or $(1+2\pi)$ rad or $(1+4\pi)$ rad, etc. By the same logic (or by doing the unit conversion - try it if you're unsure!), they will have the same phase difference if the difference in number of wavelengths is $\frac{1}{2\pi}$ wavelengths or $\left(1+\frac{1}{2\pi}\right)$ wavelengths or $\left(2+\frac{1}{2\pi}\right)$ wavelengths. So for the second smallest length, we would have set the equation

$$1+\frac{1}{2\pi} = \frac{L}{(430\times10^{-9}\,m)}(1.55-1.37)$$

I was going to give another example, but it would have essentially been 35.9. Do that problem!

Double-Slit Interference

Let's start with what we had in the chart.

The path length is $d\sin\theta$. Since the waves begin in phase, $d\sin\theta = m\lambda$ gives constructive interference and $d\sin\theta = (m+\frac{1}{2})\lambda$ gives destructive interference.

Here's a problem to get us started:

Question: Two slits separated by 57 μm are illuminated by light of wavelength 589 nm. What is the angular separation between the 4th and 5th dark fringes?

Solution: The first dark fringe is $m=0$. So the 4th and 5th are $m=3$ and $m=4$, respectively. Since we want dark fringes, we want

$$d\sin\theta = (m+\tfrac{1}{2})\lambda$$

The given variables are

$$d = 57\times10^{-6}\,m$$
$$\lambda = 589\times10^{-9}\,m$$

We're asked to find the angular separation

$$\theta_4 - \theta_3$$

We can find θ_3 and θ_4 from $d\sin\theta = (m+\frac{1}{2})\lambda$

$$d\sin\theta_3 = (3+\tfrac{1}{2})\lambda$$
$$(57\times10^{-6}\,m)\sin\theta_3 = (3.5)(589\times10^{-9}\,m)$$
$$\theta_3 = 0.036\,rad$$

$$d\sin\theta_4 = (4+\tfrac{1}{2})\lambda$$
$$(57\times10^{-6}\,m)\sin\theta_4 = (4.5)(589\times10^{-9}\,m)$$
$$\theta_4 = 0.047\,rad$$

$$\theta_4 - \theta_3 = 0.047\,rad - 0.036\,rad = \boxed{0.011\,rad}$$

You can answer in degrees instead; that's fine. But if you decide to use the small angle approximation

$$\sin\theta \approx \theta$$

then your answer will come out in radians.

Here's another:

Question: If the angular deviation of the $m=2$ bright fringe of light of wavelength 619nm through a double-slit is 0.9°, what is the separation between the slits?

Solution: The given variables are

$m=2$

$\lambda = 619\times10^{-9}\,m$

$\theta = 0.9^\circ$

We want the distance between the slits, d, so this is a case of plug and chug.

$d\sin\theta = m\lambda$ (bright fringes are the result of constructive interference)

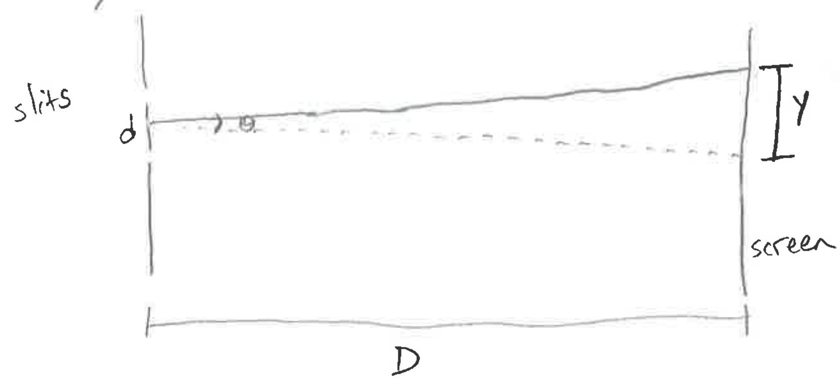
$$d = \frac{m\lambda}{\sin\theta}$$

$$= \frac{2(619\times10^{-9}\,m)}{\sin(0.9^\circ)}$$

Make sure your calculator is in degree mode!

$$\boxed{\begin{aligned} d &= 88.7\times10^{-6}\,m \\ &= 88.7\,\mu m \end{aligned}}$$

Now let's turn our attention to the details of the double-slit problem. Make sure you understand the geometry of the problem. Here is a rough sketch.



$\tan\theta = \frac{y}{D}$

Because the screen is very far away (D is very large), we can use the small angle approximation:

$\sin\theta \sim \tan\theta \sim \theta$

But this approximation is not an excuse for ignoring the geometry!

Let's try a problem that tests our understanding of the details:

Question: Monochromatic light illuminates a double-slit and forms an interference pattern on a screen 2.7m away. If the fifth-order bright fringe is 13.8cm from the central bright fringe, and the distance between the slits is 60um, what is the wavelength of the light?

Solution: Since we're dealing with bright fringes, we need $d\sin\theta = m\lambda$, not $d\sin\theta = (m+\frac{1}{2})\lambda$. We're also given the distance to the screen and the y-distance on the screen, utilized in $\tan\theta = \frac{y}{D}$.

$D = 2.7m$

$m = 5$

$y_5 = 13.8cm = 0.138m$

$d = 60\mu m = 60\times10^{-6}m$

$\tan\theta = \frac{y}{D}$

$d\sin\theta = m\lambda$

small angle approximation

$\sin\theta \sim \tan\theta \sim \theta$

$\theta = \frac{y}{D}$

$d\theta = m\lambda$

$d\left(\frac{y}{D}\right) = m\lambda$

$\lambda = \frac{dy}{Dm}$

$= \frac{(60\times10^{-6}m)(0.138m)}{(2.7m)5}$

$\boxed{\lambda = 613nm}$

In this problem it's not actually necessary to use the small angle approximation; you could solve for θ using arctan and then plug it into $d\sin\theta = m\lambda$.

Another example of the geometrical details:

Question: In a double-slit experiment, the distance between the slits is 100um and the distance from the slits to the screen is 95cm. If the wavelength of light is 589nm, which bright fringe is visible 1.12cm from the central bright fringe?

Solution: Again, bright fringes mean we use $d\sin\theta = m\lambda$ instead of $d\sin\theta = (m+\frac{1}{2})\lambda$.
We don't have θ, but we can get it from $\tan\theta = \frac{y}{D}$.

$d = 100\,\mathrm{um} = 100\times10^{-6}\,\mathrm{m}$
$D = 95\,\mathrm{cm} = 0.95\,\mathrm{m}$
$\lambda = 589\,\mathrm{nm} = 589\times10^{-9}\,\mathrm{m}$
$y = 1.12\,\mathrm{cm} = 0.0112\,\mathrm{cm}$

$d\sin\theta = m\lambda$

$\tan\theta = \frac{y}{D}$

small angle approximation
$\sin\theta \sim \tan\theta \sim \theta$

$d\theta = m\lambda$

$\theta = \frac{y}{D}$

$d\left(\frac{y}{D}\right) = m\lambda$

$$m = \frac{dy}{D\lambda}$$

$$= \frac{(100\times10^{-6}\,\mathrm{m})(0.0112\,\mathrm{m})}{(0.95\,\mathrm{m})(589\times10^{-9}\,\mathrm{m})}$$

$\boxed{m = 2}$

So the second-order fringe is the one 1.12cm from the central maximum.

Like the previous problem, we don't need to use the small angle approximation to solve the problem.

One more:

Question: What is the separation on the screen of the first and third dark fringes caused by a double-slit experiment with slit separation $45\mu m$, slit-to-screen distance $1.0m$, and light of wavelength $480nm$?

Answer: We're talking about dark fringes, which are given by $d\sin\theta=(m+\frac{1}{2})\lambda$. We also have the first and third fringes, which for destructive interference means $m=0$ and $m=2$. We're asked to find the distance y_1-y_0, so we need $\tan\theta=\frac{y}{D}$.

$d=45\mu m=45\times10^{-6}m$
$D=1.0m$
$\lambda=480nm=480\times10^{-9}m$

$$d\sin\theta=(m+\tfrac{1}{2})\lambda \qquad \tan\theta=\frac{y}{D}$$

small angle approximation
$\sin\theta\sim\tan\theta\sim\theta$

$$d\theta=(m+\tfrac{1}{2})\lambda \qquad \theta=\frac{y}{D}$$

$$d\left(\frac{y}{D}\right)=(m+\tfrac{1}{2})\lambda$$

$$y=\frac{(m+\frac{1}{2})\lambda D}{d}$$

$$y_2-y_0=\frac{(2+\frac{1}{2})\lambda D}{d}-\frac{(0+\frac{1}{2})\lambda D}{d}$$

$$=\frac{\lambda D}{d}\left[2+\tfrac{1}{2}-(0+\tfrac{1}{2})\right]$$

$$=\frac{2\lambda D}{d}$$

$$=\frac{2(480\times10^{-9}m)(1.0m)}{45\times10^{-6}m}$$

$$\boxed{y_2-y_0=0.021m=2.1cm}$$

Once again, we didn't actually need the small angle approximation.

Of course, there is another set of details we should know about the double-slit experiment, and those details are diffraction – what happens when we take diffraction into account in this experiment. We will wait to address that until the diffraction section.

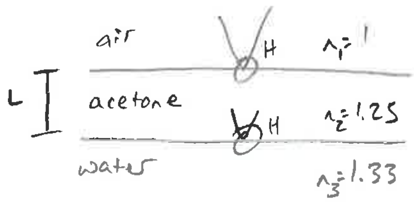
For now, we will move on to...

Thin Films

If we consider the "details" of the thin film problem to be identifying hard and soft reflections, we can't separate those details from the main problem because we need them to set up the problem. So we'll consider everything together.

Question: Light of wavelength 545nm is incident normal to a layer of acetone ($n=1.25$) on water ($n=1.33$). If the acetone appears black, what is the minimum possible thickness of the acetone?

Solution:



Here both of our reflections are hard ($n_2 > n_1$ and $n_3 > n_2$). Both waves experience a π phase change, so the waves are in phase with each other.

$\lambda = 545\,nm = 545 \times 10^{-9}\,m$

Since the waves are in phase, we have:

$2L = m\lambda$ constructive $\qquad 2L = (m+\frac{1}{2})\lambda$ destructive

Since the thin film appears black, we want destructive interference.

$$2L = (m+\tfrac{1}{2})\lambda$$

$$2L = (m+\tfrac{1}{2})(545\times10^{-9}\,m)$$

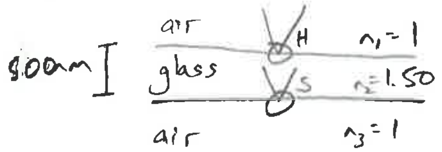
We also want the minimum possible thickness. L will be smallest if $m=0$.

$$2L = (0+\tfrac{1}{2})(545\times10^{-9}\,m)$$

$$\boxed{L = 136\,nm}$$

Question: A 500-nm-thick glass plate held in air is illuminated by sunlight. What are the visible wavelengths (380 - 750nm) that will interfere constructively when reflected from the plate?

Solution:



Here the first reflection is hard ($n_2 > n_1$) and the second is soft ($n_3 < n_2$). The wave undergoing the hard reflection experiences a π phase shift, but the wave undergoing the soft reflection experiences no phase shift. So the two reflected waves are π out of phase with each other.

Since the waves are out of phase

$2L = m\lambda$ destructive $\qquad$ $2L = (m + \frac{1}{2})\lambda$ constructive

We want constructive interference

$$2L = (m + \tfrac{1}{2})\lambda$$

Now we want to find the wavelengths, so let's begin plugging in values for m.

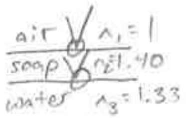
$m = 0$	$2(500\times10^{-9}m) = (0+\frac{1}{2})\lambda$	$\lambda_0 = 2\times10^{-6}m = 2000nm$
$m = 1$	$2(500\times10^{-9}m) = (1+\frac{1}{2})\lambda$	$\lambda_1 = 6.67\times10^{-7}m = 667nm$
$m = 2$	$2(500\times10^{-9}m) = (2+\frac{1}{2})\lambda$	$\lambda_2 = 4\times10^{-7}m = 400nm$
$m = 3$	$2(500\times10^{-9}m) = (3+\frac{1}{2})\lambda$	$\lambda_3 = 2.86\times10^{-7}m = 286nm$

The values between 380nm - 750nm are

$$\boxed{\begin{aligned}\lambda_2 &= 400nm \\ \lambda_1 &= 667nm\end{aligned}}$$

Question: 300nm of soap is resting on 20cm of water, which in turn is enclosed in a glass tank whose thickness is 15mm. If the top of the tank is removed and light of wavelength 400nm is shown, will the interference be constructive or destructive? (The index of refraction of soap is $n = 1.40$, water is $n = 1.33$ and glass is 1.50.)

Solution: The key here is identifying the thin film. At first it might seem like water is the thin film. After all, water is between soap and glass - it's in the middle, so shouldn't it be n_2? The problem with that line of thinking is that the glass tank is probably on a table, which might be on carpet, on concrete, on the earth's crust. So picking "the middle" can prove problematic, and identifying the thin film requires thinking through the situation. In this situation, there is only one layer that is thin: the soap.



Now we have a hard reflection ($n_2 > n_1$) and a soft reflection ($n_3 < n_2$). With one phase shift of π and one phase shift of zero, the waves are out of phase.

$2L = m\lambda$ destructive $\qquad$ $2L = (m + \frac{1}{2})\lambda$ constructive

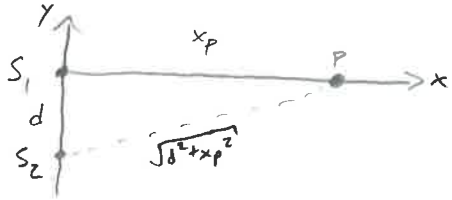
We're not sure which case we want, so let's start by plugging in what we do know.

$$2(300 \times 10^{-9} m) = ? \, (400 \times 10^{-9} m)$$

The missing piece is 1.5, so it must be $m + \frac{1}{2}$, where $m = 1$. And in this problem, $m + \frac{1}{2}$ corresponds to $\boxed{\text{constructive interference}}$.

Two Sources

35.25 from your homework is a good example of the 2 sources problem. To emphasize how we choose m (it's not arbitrary!), let's do #25 except we'll find the smallest positive value of x_p for destructive interference.



$d = 2.70\,\mu m = 2.70\times10^{-6}\,m$
$\lambda = 900\,nm = 900\times10^{-9}\,m$

To review our solution for the actual #25, the path length for source S_1 is just x_p, and using the Pythagorean Theorem, the path length for source S_2 is $\sqrt{d^2+x_p^2}$. So the path length difference is $\sqrt{d^2+x_p^2} - x_p$, and since the sources emit in phase

$$\sqrt{d^2+x_p^2} - x_p = \left(m+\tfrac{1}{2}\right)\lambda \qquad \text{destructive}$$

$$\sqrt{d^2+x_p^2} = x_p + \left(m+\tfrac{1}{2}\right)\lambda$$

$$d^2 + \cancel{x_p^2} = \cancel{x_p^2} + \left(m+\tfrac{1}{2}\right)^2\lambda^2 + 2x_p\left(m+\tfrac{1}{2}\right)\lambda$$

$$d^2 - \left(m+\tfrac{1}{2}\right)^2\lambda^2 = 2x_p\left(m+\tfrac{1}{2}\right)\lambda$$

$$\frac{d^2}{2\left(m+\frac{1}{2}\right)\lambda} - \frac{\left(m+\frac{1}{2}\right)\lambda}{2} = x_p$$

We know $m=0$ gives us maximum x_p no matter what d and λ are, but the minimum positive x_p should depend on all our variables.

$m=1$ $$\frac{(2.70\times10^{-6}\,m)^2}{2\left(1+\frac{1}{2}\right)(900\times10^{-9}\,m)} - \frac{\left(1+\frac{1}{2}\right)(900\times10^{-9}\,m)}{2} = 2.03\times10^{-6}\,m$$

$m=2$ $$\frac{(2.70\times10^{-6}\,m)^2}{2\left(2+\frac{1}{2}\right)(900\times10^{-9}\,m)} - \frac{\left(2+\frac{1}{2}\right)(900\times10^{-9}\,m)}{2} = 4.95\times10^{-7}\,m$$

$m=3$ $$\frac{(2.70\times10^{-6}\,m)^2}{2\left(3+\frac{1}{2}\right)(900\times10^{-9}\,m)} - \frac{\left(3+\frac{1}{2}\right)(900\times10^{-9}\,m)}{2} = -4.18\times10^{-7}\,m$$

So the smallest positive value of x_p is $\boxed{495\,nm}$.